Application of the Exp-Function Method to the Riccati Equation and New Exact Solutions with Three Arbitrary Functions of Quantum Zakharov Equations

Mohamed A. Abdou[a,b] and Essam M. Abulwafa[a]

[a] Theoretical Research Group, Physics Department, Faculty of Science, Mansoura University, 35516 Mansoura, Egypt
[b] Faculty of Education for Girls, Physics Department, King Kahlid University, Bisha, Saudi Arabia

Reprint requests to M. A. A.; E-mail: m_abdou_eg@yahoo.com

Z. Naturforsch. **63a,** 646 – 652 (2008); received March 31, 2008

The Exp-function method with the aid of the symbolic computational system is used for constructing generalized solitary solutions of the generalized Riccati equation. Based on the Riccati equation and its generalized solitary solutions, new exact solutions with three arbitrary functions of quantum Zakharov equations are obtained. It is shown that the Exp-function method provides a straightforward and important mathematical tool for nonlinear evolution equations in mathematical physics.

Key words: Exp-Function Method; Generalized F-Expansion Method; Quantum Zakharov Equations; New Solitary and Periodic Solutions.

1. Introduction

The investigation of the travelling wave solutions of nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. The nonlinear evolution equations are major subjects in physical science, appearing in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, and chemical physics. Nonlinear wave phenomena of dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. In the past several decades, new exact solutions helped to find new phenomena. A variety of powerful methods for obtaining the exact solutions of nonlinear evolution equations has been presented [1 – 33].

More recently, He and Abdou [22], and El-Wakil et al. [23, 25] proposed a straightforward and concise method, called Exp-function method, to obtain generalized solitary solutions and periodic solutions; applications of the method for solving nonlinear evolution equations arising in mathematical physics can be found in [22 – 26]. The solution procedure of this method, with the aid of Maple, is of utter simplicity, and this method can easily be extended to other kinds of nonlinear evolution equations.

Here, the Exp-function method is proposed for seeking new generalized solitary solutions of the generalized Riccati equation

$$\phi' = q + r\phi(\xi) + p\phi^2(\xi), \tag{1}$$

where p, q and r are constants to be determined later. Then (1) and its generalized solitary solutions are employed to find new and more general exact solutions of the quantum Zakharov equations [34, 35]

$$\begin{aligned} &\mathrm{i}\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial x^2} - H^2\frac{\partial^4 E}{\partial x^4} = nE,\\ &\frac{\partial^2 n}{\partial t^2} - \frac{\partial^2 n}{\partial x^2} + H^2\frac{\partial^4 n}{\partial x^4} = \frac{\partial^2 |E|^2}{\partial x^2}, \end{aligned} \tag{2}$$

where E is the electric field, n the plasma density, and H the dimensionless quantum parameter,

$$H = \frac{\hbar\omega}{k_\mathrm{B}T_\mathrm{e}}.$$

The quantum parameter H given in (2) expresses the ratio between the ion plasma energy and the electron thermal energy. If we set $\hbar = 0$, i. e. $H = 0$, we simply obtain the classical model. At the classical level, a set of coupled nonlinear wave equations describing the interaction between high frequency Langmuir waves and low frequency ion acoustic waves was first derived by Zakharov [34, 35].

2. The Exp-Function Method for the Generalized Riccati Equation

Introducing a complex variable η defined as

$$\eta = k\xi + w,$$

where k is a constant to be determined later and w is an arbitrary constant, (1) becomes

$$k\phi' - q - r\phi - p\phi^2 = 0. \quad (3)$$

According to the Exp-function method [22–26], we assume that the solution of (3) can be expressed as

$$\phi(\eta) = \frac{\sum_{n=-c}^{d} a_n \exp(n\eta)}{\sum_{m=-f}^{g} b_m \exp(m\eta)}, \quad (4)$$

where c, d, f and g are positive integers unknown and to be determined later, a_n and b_m are unknown constants. Equation (4) can be re-written as

$$\phi(\eta) = \frac{a_c \exp(c\eta) + \ldots + a_{-d} \exp(-d\eta)}{b_f \exp(f\eta) + \ldots + b_{-g} \exp(-g\eta)}. \quad (5)$$

In order to determine values of c and f, we balance the linear term of the highest-order in (3) with the highest-order nonlinear term ϕ' and ϕ^2. Then we have

$$\phi' = \frac{c_1 \exp[(f+c)\eta] + \ldots}{c_2 \exp[2f\eta] + \ldots}, \quad (6)$$

$$\phi^2 = \frac{c_3 \exp[2c\eta] + \ldots}{c_4 \exp[2f\eta] + \ldots}, \quad (7)$$

where c_i are coefficients for simplicity. By balancing the highest order of the Exp-function in (6) and (7), we have

$$c + f = 2c, \quad (8)$$

which leads to the results $f = c$. Proceeding in the same manner as illustrated above, we can determine values of d and g. Balancing the linear term of lowest order in (3), we have

$$\phi' = \frac{d_1 \exp[-(g+d)\eta] + \ldots}{d_2 \exp[-2g\eta] + \ldots}, \quad (9)$$

$$\phi^2 = \frac{d_3 \exp[-2d\eta] + \ldots}{d_4 \exp[-2g\eta] + \ldots}, \quad (10)$$

where d_i are coefficients for simplicity. By balancing the highest order of the Exp-function in (9) and (10), we have

$$-(g+d) = -2d, \quad (11)$$

which leads to the result $g = d$. We can freely choose the values of c and d, but the final solution does not strongly depend upon the choice of values of c and d [22]. For simplicity, we set $f = c = 1$ and $d = g = 1$, then (5) becomes

$$\phi(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)}. \quad (12)$$

Substituting (12) into (3), we have

$$\begin{aligned}
&\frac{1}{A}\big[C_2 \exp(2\eta) + C_1 \exp(\eta) + C_0 \\
&\quad + C_{-1} \exp(-\eta) + C_{-2} \exp(-2\eta)\big] = 0, \\
&A = \big[b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)\big]^2, \\
&C_2 = -pa_1^2 - qb_1^2 - ra_1 b_1, \\
&C_1 = ka_1 b_0 - ka_0 b_1 - 2qb_1 b_0 - 2pa_1 a_0 \\
&\quad - ra_1 b_0 - ra_0 b_1, \\
&C_0 = -ra_0 b_0 - 2pa_1 a_{-1} - 2ka_{-1} b_1 \\
&\quad - qb_0^2 - ra_{-1} b_1 + 2ka_1 b_{-1} \\
&\quad - pa_0^2 - ra_1 b_{-1} - 2qb_1 b_{-1}, \\
&C_{-1} = -ka_{-1} b_0 + ka_0 b_{-1} - 2qb_0 b_{-1} \\
&\quad - 2pa_0 a_{-1} - ra_0 b_{-1} - ra_{-1} b_0, \\
&C_{-2} = -ra_{-1} b_{-1} - qb_{-1}^2 - pa_{-1}^2.
\end{aligned} \quad (13)$$

Equating the coefficients of $\exp(j\eta)$ $(j = 2, 1, 0, -1, -2)$ to zero, we have

$$C_{-2} = 0, C_1 = 0, C_0 = 0, C_2 = 0, C_{-1} = 0.$$

Solving this system of algebraic equations with the aid of Maple, we obtain the following cases.

Case A.

$$\begin{aligned}
&k = 1, \quad b_0 = 0, \quad a_0 = 0, \quad b_1 = b_1, \\
&a_{-1} = a_{-1}, \quad b_{-1} = -\frac{a_{-1}(r+2)}{2q}, \\
&a_1 = -\frac{2qb_1}{-2+r}, \quad p = \frac{(r+2)(2+r)}{4q}.
\end{aligned} \quad (14)$$

Case B.

$$k=-\mathrm{i},\quad b_0=0,\quad a_0=0,\quad b_1=b_1,\\ a_{-1}=a_{-1},\quad b_{-1}=-\frac{a_{-1}(r-2\mathrm{i})}{2q},\\ a_1=-\frac{2qb_1}{2\mathrm{i}+r},\quad p=\frac{(r-2\mathrm{i})(2\mathrm{i}+r)}{4q}. \tag{15}$$

Case C.

$$k=1,\quad b_0=b_0,\quad a_0=a_0,\quad b_1=b_1,\\ a_1=-\frac{2qb_1}{-1+r},\quad p=\frac{r^2-1}{4q},\\ a_{-1}=\big[a_0^2r^3-r^2a_0^2-ra_0^2+a_0^2+4qa_0b_0r^2\\ -4qra_0b_0+4rq^2b_0^2-4q^2b_0^2\big]/8qb_1,\\ b_{-1}=\big(a_0r^2-a_0+2qb_0+2rqb_0\big)\\ \cdot\big(-2qb_0+r^2a_0-a_0+2rqb_0\big)/16q^2b_1. \tag{16}$$

Substituting (14) into (12), we obtain the following generalized solitary solution of (1):

$$\phi(\eta)=\frac{-\frac{2qb_1}{-2+r}\exp(\eta)+a_{-1}\exp(-\eta)}{b_1\exp(\eta)-\frac{a_{-1}(r+2)}{2q}\exp(-\eta)},\\ \eta=\xi+w. \tag{17}$$

If we set $a_{-1}=-q^2$, $b_1=1$, $r=1$ and $w=0$, (17) reduces to

$$\phi(\eta)=2q\frac{2\exp(\xi)-q\exp(-\xi)}{2\exp(\xi)+3q\exp(-\xi)}. \tag{18}$$

For $a_{-1}=q^2$, $b_1=1$, $r=1$ and $w=0$, (17) yields

$$\phi(\eta)=2q\frac{2\exp(\xi)+q\exp(-\xi)}{2\exp(\xi)-3q\exp(-\xi)}. \tag{19}$$

Substituting (15) into (12), admits the new generalized solitary solution of (1)

$$\phi(\eta)=\frac{-\frac{2qb_1}{2\mathrm{i}+r}\exp(\eta)+a_{-1}\exp(-\eta)}{b_1\exp(\eta)+\frac{a_{-1}(r-2\mathrm{i})}{2q}\exp(-\eta)},\\ \eta=-\mathrm{i}\xi+w. \tag{20}$$

If we set $a_{-1}=1$, $b_1=\mathrm{i}$, $r=1$ and $w=0$, (20) becomes

$$\phi(\eta)=-2q\big[2q\mathrm{i}\exp(-\mathrm{i}\xi)-2\mathrm{i}\exp(\mathrm{i}\xi)\\ -\exp(\mathrm{i}\xi)\big]\big[(2\mathrm{i}+1)(2q\mathrm{i}\exp(-\mathrm{i}\xi)\\ -\exp(\mathrm{i}\xi)+2\mathrm{i}\exp(\mathrm{i}\xi))\big]^{-1}. \tag{21}$$

If we set $a_{-1}=-1$, $b_1=\mathrm{i}$, $r=1$ and $w=0$, then (20) becomes

$$\phi(\eta)=-2q\big[2q\mathrm{i}\exp(-\mathrm{i}\xi)+2\mathrm{i}\exp(\mathrm{i}\xi)\\ +\exp(\mathrm{i}\xi)\big]\big[(2\mathrm{i}+1)(2q\mathrm{i}\exp(-\mathrm{i}\xi)\\ +\exp(\mathrm{i}\xi)-2\mathrm{i}\exp(\mathrm{i}\xi))\big]^{-1}. \tag{22}$$

Substituting (16) into (12), admits the following generalized solitary solution of (1):

$$\phi(\eta)=\Big\{-\frac{2qb_1}{r-1}\exp(\eta)+a_0+\big(a_0^2r^3-r^2a_0^2\\ -ra_0^2+a_0^2+4qa_0b_0r^2-4qra_0b_0\\ +4rq^2b_0^2-4q^2b_0^2\big)\exp(-\eta)/8qb_1\Big\}\Big/\\ \Big\{b_1\exp(\eta)+b_0+\big(a_0r^2-a_0+2qb_0\\ +2rqb_0\big)\big(-2qb_0+r^2a_0-a_0\\ +2rqb_0\big)\exp(-\eta)/16q^2b_1\Big\},\\ \eta=\xi+w. \tag{23}$$

If we set $a_0=4\mathrm{i}q\sqrt{q}$, $b_1=1$, $r=2$, $b_0=0$ and $w=0$, then (23) reduces to

$$\phi(\eta)=-2q\frac{(\exp(\xi)-2\mathrm{i}\sqrt{q}+3q\exp(-\xi))}{(\exp(\xi)+9q\exp(-\xi))}. \tag{24}$$

In case of $a_0=4q\sqrt{2q}$, $b_1=1$, $r=2$, $b_0=2\sqrt{q}$ and $w=0$, (23) reduces to

$$\phi(\eta)=-2q\big(-\exp(\xi)+2\sqrt{2q}+7q\exp(-\xi)\\ +4q\sqrt{2}\exp(-\xi)\big)\big(-\exp(\xi)\\ -2\sqrt{q}+21q\exp(-\xi)\\ +12q\sqrt{2}\exp(-\xi)\big). \tag{25}$$

It has to be noted, that the solutions obtained in (17), (20) and (23) are exactly the same as those obtained in [24] in case of $r=0$.

3. Exact Solutions of the Quantum Zakharov Equations

To look for the travelling wave solutions of (2), we make the ansatz

$$E(x,t)=U(\xi)\exp[\mathrm{i}(kx+ct+\delta_1)],\\ \xi=x+t+\delta_2,\quad n(x,t)=N(\xi), \tag{26}$$

where k, c, δ_1 and δ_2 are constants to be determined. Substituting (26) into (2) and separating the real and imaginary parts lead to

$$(1+2k+4H^2k^3)U' - 4H^2kU''' = 0, \tag{27}$$

$$\begin{aligned}&(-c-k^2-H^2k^4-N)U\\&+(1+6H^2k^2)U'' = H^2U'''',\end{aligned} \tag{28}$$

$$H^2N'''' - 2U'^2 - 2UU'' = 0. \tag{29}$$

Inserting the differential of (27) into (28) gives

$$\begin{aligned}&[-c-k^2-H^2k^4-N]U\\&+\left[1+6H^2k^2-\frac{1}{4k}(1+2k+4H^2k^3)\right]U'' = 0.\end{aligned} \tag{30}$$

Using the generalized F-expansion method, we suppose that the solutions of (29) and (30) can be expressed by

$$U(\xi) = \sum_{i=0}^{M_1} k_i\phi^i(\xi) + \sum_{i=0}^{M_1} h_i\phi^{-i}(\xi), \tag{31}$$

$$N(\xi) = \sum_{i=0}^{M_2} c_i\phi^i(\xi) + \sum_{i=0}^{M_2} d_i\phi^{-i}(\xi). \tag{32}$$

Balancing the highest linear terms with the highest nonlinear terms in (29) and (30), we find $M_1 = 2$, $M_2 = 2$ and

$$\begin{aligned}U(\xi) = {}& k_0 + k_1\phi(\xi) + k_2\phi^2(\xi) + h_0\\&+ h_1\phi^{-1}(\xi) + h_2\phi^{-2}(\xi),\end{aligned} \tag{33}$$

$$\begin{aligned}N(\xi) = {}& c_0 + c_1\phi(\xi) + c_2\phi^2(\xi) + d_0\\&+ d_1\phi^{-1}(\xi) + d_2\phi^{-2}(\xi).\end{aligned} \tag{34}$$

Substituting (33) and (34) into (29) and (30) along with (1), and setting each coefficients of $\phi^{\mathrm{i}}(\xi)$ to zero, we can deduce the set of algebraic equations for k_0, k_1, k_2, h_0, h_1, h_2, c_0, c_1, c_2, d_0, d_1 and d_2. Solving the set of algebraic equations, we can distinguish the following different cases:

Case 1. When $q = 0$, $r = 1$, $p = -1$, then

$$k = k, \quad c = c, \quad d_0 = d_0, \quad h_0 = h_0, \quad h_1 = 0,$$

$$d_1 = 0, \quad c_1 = -\frac{3}{2}\frac{20k^3H^2-1+2k}{k},$$

$$d_2 = -\frac{1}{160}\frac{20k^3H^2-1+2k}{k},$$

$$c_2 = \frac{3}{2}\frac{20k^3H^2-1+2k}{k},$$

$$k_2 = \sqrt{-\frac{-180k^3H^2+9-18k}{k}}H,$$

$$k_1 = -\sqrt{-\frac{-180k^3H^2+9-18k}{k}}H,$$

$$\begin{aligned}c_0 = -\frac{1}{8k}\big(&8k^5H^2-20H^2k^3+8k^3-2k+8ck\\&+1+8kd_0\big),\end{aligned}$$

$$k_0 = -\frac{3H-6Hk-60k^3H^3+4b_0\sqrt{-\frac{-180k^3H^2+9-18k}{k}}k}{4\sqrt{-\frac{-180k^3H^2+9-18k}{k}}k},$$

$$h_2 = -\frac{3}{80}\frac{H(20H^2k^3-1+2k)}{\sqrt{\frac{-180k^3H^2+9-18k}{k}}k}. \tag{35}$$

Case 2. When $q = 0$, $r = -1$, $p = 1$, then

$$k = k, \quad c = c, \quad d_0 = d_0, \quad h_0 = h_0, \quad h_1 = 0,$$

$$d_1 = 0, \quad c_1 = -\frac{3}{2}\frac{20k^3H^2-1+2k}{k},$$

$$d_2 = -\frac{1}{160}\frac{20k^3H^2-1+2k}{k},$$

$$c_2 = \frac{3}{2}\frac{20k^3H^2-1+2k}{k},$$

$$k_2 = \sqrt{-\frac{-180k^3H^2+9-18k}{k}}H,$$

$$k_1 = -\sqrt{-\frac{-180k^3H^2+9-18k}{k}}H,$$

$$c_0 = -\frac{8k^5H^2-20H^2k^3+8k^3-2k+8ck+1+8kd_0}{8k},$$

$$k_0 = -\frac{3H-6Hk-60k^3H^3+4b_0\sqrt{-\frac{-180k^3H^2+9-18k}{k}}k}{4k\sqrt{-\frac{-180k^3H^2+9-18k}{k}}k},$$

$$h_2 = -\frac{3}{80}\frac{H(20H^2k^3-1+2k)}{k\sqrt{\frac{-180k^3H^2+9-18k}{k}}k}. \tag{36}$$

Case 3. When $q = \frac{1}{2}$, $r = 0$, $p = -\frac{1}{2}$, then

$$c_0 = c_0, \quad k = k, \quad c = c, \quad k_1 = 0,$$

$$d_0 = -\frac{4k^5H^2+20H^2k^3+4k^3+2k+4ck-1+4kc_0}{4k},$$

$$h_0 = h_0, \quad h_1 = 0, \quad d_1 = 0, \quad c_1 = 0,$$

$$d_2 = -\frac{1}{40}\frac{20k^3H^2 - 1 + 2k}{k},$$

$$c_2 = \frac{3}{8}\frac{20k^3H^2 - 1 + 2k}{k},$$

$$k_2 = \frac{1}{4}\sqrt{-\frac{-180k^3H^2 - 9 - 18k}{k}}H,$$

$$k_0 = -\frac{-3H + 6Hk + 60k^3H^3 + 2b_0\sqrt{-\frac{-180k^3H^2-9-18k}{k}k}}{2\sqrt{-\frac{-180k^3H^2-9-18k}{k}k}},$$

$$h_2 = -\frac{3}{20}\frac{H(20H^2k^3 - 1 + 2k)}{\sqrt{-\frac{-180k^3H^2-9-18k}{k}k}}. \quad (37)$$

Case 4. When $q = 1$, $r = 0$, $p = -1$, then

$$c_1 = 0, \quad k = k, \quad c = c, \quad k_1 = 0, \quad k_0 = k_0,$$

$$d_0 = d_0, \quad d_1 = 0, \quad h_1 = 0,$$

$$c_0 = -\frac{k^5H^2 + 20H^2k^3 + k^3 + 2k + ck - 1 + kd_0}{k},$$

$$c_2 = \frac{3}{2}\frac{2k - 1 + 20H^2k^3}{k},$$

$$d_2 = -\frac{1}{10}\frac{20k^3H^2 - 1 + 2k}{k},$$

$$k_2 = \sqrt{-\frac{-180k^3H^2 + 9 - 18k}{k}}H,$$

$$h_0 = \frac{-6H + 12Hk + 120k^3H^3 + a_0\sqrt{-\frac{-180k^3H^2+9-18k}{k}k}}{\sqrt{-\frac{-180k^3H^2+9-18k}{k}k}},$$

$$h_2 = -\frac{3}{5}\frac{H(20H^2k^3 - 1 + 2k)}{-\sqrt{-\frac{-180k^3H^2+9-18k}{k}k}}. \quad (38)$$

Case 5. When $q = 1$, $r = 0$, $p = \frac{1}{2}$, then

$$h_0 = h_0, \quad c_1 = 0, \quad k = k, \quad c = c, \quad k_1 = 0,$$

$$d_0 = d_0, \quad d_1 = 0, \quad h_1 = 0,$$

$$c_0 = -\frac{4k^5H^2 - 20H^2k^3 + 4k^3 + 4ck - 2k + 1 + 4kd_0}{4k},$$

$$c_2 = \frac{3}{8}\frac{20k^3H^2 - 1 + 2k}{k},$$

$$d_2 = -\frac{1}{40}\frac{20k^3H^2 - 1 + 2k}{k},$$

$$k_2 = \frac{1}{4}\sqrt{-\frac{-180k^3H^2 + 9 - 18k}{k}}H,$$

$$k_0 = \frac{-3H + 6Hk + 60k^3H^3 - 2b_0\sqrt{-\frac{-180k^3H^2+9-18k}{k}k}}{2\sqrt{-\frac{-180k^3H^2+9-18k}{k}k}},$$

$$h_2 = -\frac{3}{20}\frac{H(20H^2k^3 - 1 + 2k)}{\sqrt{-\frac{-180k^3H^2+9-18k}{k}k}}. \quad (39)$$

Case 6. When $q = -\frac{1}{2}$, $r = 0$, $p = -\frac{1}{2}$, then

$$h_0 = h_0, \quad c_1 = 0, \quad k = k, \quad c = c, \quad k_1 = 0,$$

$$c_0 = -\frac{4k^5H^2 - 20H^2k^3 + 4k^3 - 2k + 4ck + 1 + 4kd_0}{4k},$$

$$c_2 = \frac{3}{8}\frac{2k - 1 + 20H^2k^3}{k},$$

$$d_0 = d_0, \quad d_1 = 0, \quad h_1 = 0,$$

$$d_2 = -\frac{1}{40}\frac{20k^3H^2 - 1 + 2k}{k},$$

$$k_2 = \frac{1}{4}\sqrt{-\frac{-180k^3H^2 + 9 - 18k}{k}}H,$$

$$k_0 = \frac{-3H + 6Hk + 60k^3H^3 - 2b_0\sqrt{-\frac{-180k^3H^2+9-18k}{k}k}}{2\sqrt{-\frac{-180k^3H^2+9-18k}{k}k}},$$

$$h_2 = -\frac{3}{20}\frac{H(20H^2k^3 - 1 + 2k)}{-\sqrt{-\frac{-180k^3H^2+9-18k}{k}k}}. \quad (40)$$

Case 7. When $q = 1$, $r = 0$, $p = 1$, then

$$h_1 = 0, \quad c_1 = 0, \quad k = k, \quad c = c,$$

$$c_0 = c_0, \quad d_1 = 0,$$

$$c_2 = \frac{3}{2}\frac{2k - 1 + 20H^2k^3}{k},$$

$$h_0 = \frac{2H}{3}\sqrt{\frac{18k - 9 + 180H^2k^3}{k}},$$

$$d_0 = -\frac{k^5H^2 - 20H^2k^3 + k^3 + c_0k + ck + 1 - 2k}{k},$$

$$k_1 = 0, \quad d_2 = -\frac{1}{10}\frac{20k^3H^2 - 1 + 2k}{k},$$

$$k_2 = \sqrt{-\frac{-180k^3H^2 + 9 - 18k}{k}}H,$$

$$k_0=\frac{6H-12Hk-120k^3H^3+b_0\sqrt{-\frac{-180k^3H^2+9-18k}{k}k}}{\sqrt{-\frac{-180k^3H^2+9-18k}{k}k}},$$

$$h_2=-\frac{3}{5}\frac{H(20H^2k^3-1+2k)}{\sqrt{-\frac{-180k^3H^2+9-18k}{k}k}}. \quad (41)$$

Case 8. When $q=0$, $r=0$, $p\neq 0$, then

$$d_1=0,\quad k_0=-k_0,\quad c_1=0,$$

$$k=k,\quad c=c,\quad h_2=h_2,$$

$$h_0=h_0,\quad c_0=c_0,\quad k_1=0,\quad h_1=0,\quad d_0=d_0,$$

$$c_2=\frac{3}{2}\frac{C^2(2k-1+20H^2k^3)}{k},$$

$$k_2=\sqrt{-\frac{-180k^3H^2+9-18k}{k}}C^2H,$$

$$d_2=-\frac{b_2(2k-1+20k^3H^3)}{-\frac{-180k^3H^2+9-18k}{Hk}}. \quad (42)$$

From (17), (35) and (26) we obtain the following new exact formal solutions of (2):

$$\begin{aligned}n_1(\xi)&=k_0+k_1\left[\frac{-\frac{2qb_1}{-2+r}\exp(\eta)+a_{-1}\exp(-\eta)}{b_1\exp(\eta)-\frac{a_{-1}(r+2)}{2q}\exp(-\eta)}\right]\\&+k_2\left[\frac{-\frac{2qb_1}{-2+r}\exp(\eta)+a_{-1}\exp(-\eta)}{b_1\exp(\eta)-\frac{a_{-1}(r+2)}{2q}\exp(-\eta)}\right]^2+h_0\\&+h_2\left[\frac{-\frac{2qb_1}{-2+r}\exp(\eta)+a_{-1}\exp(-\eta)}{b_1\exp(\eta)-\frac{a_{-1}(r+2)}{2q}\exp(-\eta)}\right]^{-2},\\E_1(\xi)&=\left[c_0+c_1\left[\frac{-\frac{2qb_1}{-2+r}\exp(\eta)+a_{-1}\exp(-\eta)}{b_1\exp(\eta)-\frac{a_{-1}(r+2)}{2q}\exp(-\eta)}\right]\right.\\&+c_2\left[\frac{-\frac{2qb_1}{-2+r}\exp(\eta)+a_{-1}\exp(-\eta)}{b_1\exp(\eta)-\frac{a_{-1}(r+2)}{2q}\exp(-\eta)}\right]^2+d_0\\&\left.+d_2\left[\frac{-\frac{2qb_1}{-2+r}\exp(\eta)+a_{-1}\exp(-\eta)}{b_1\exp(\eta)-\frac{a_{-1}(r+2)}{2q}\exp(-\eta)}\right]^{-2}\right]\\&\cdot\exp\left[\mathrm{i}(kx+ct+\delta_1)\right],\end{aligned} \quad (43)$$

where k_0, k_1, k_2, h_0, h_2, c_0, c_1, c_2, d_0 and d_2 are given by (35).

Equations (20), (35) and (26) admit the exact travelling solutions of (2) as

$$\begin{aligned}n_2(\xi)&=k_0+k_1\phi(\xi)+k_2\phi^2(\xi)+h_0+h_2\phi^{-2}(\xi),\\E_2(\xi)&=\left[c_0+c_1\phi(\xi)+c_2\phi^2(\xi)+d_0\right.\\&\left.+d_2\phi^{-2}(\xi)\right]\exp\left[\mathrm{i}(kx+ct+\delta_1)\right],\end{aligned} \quad (44)$$

where $\phi(\xi)$ is defined by (20), k_0, k_1, k_2, h_0, h_2, c_0, c_1, c_2, d_0 and d_2 are given by (35).

From (23), (35) and (26) we obtain the exact travelling solutions of (2) as

$$\begin{aligned}n_3(\xi)&=k_0+k_1\phi(\xi)+k_2\phi^2(\xi)+h_0+h_2\phi^{-2}(\xi),\\E_3(\xi)&=\left[c_0+c_1\phi(\xi)+c_2\phi^2(\xi)+d_0\right.\\&\left.+d_2\phi^{-2}(\xi)\right]\exp\left[\mathrm{i}(kx+ct+\delta_1)\right],\end{aligned} \quad (45)$$

where $\phi(\xi)$ is defined by (23), k_0, k_1, k_2, h_0, h_2, c_0, c_1, c_2, d_0 and d_2 are given by (35).

It is worth noting that, by means of Cases 2 – 8 and (17), (20) and (23), we can also obtain other exact solutions of (2). We omit them here for simplicity. To the best of our knowledge, solutions (43) – (45) have not been reported in the literature.

4. Conclusions and Discussion

We present the Exp-function method and generalized F-expansion method by computerized symbolic computation which are used for constructing new exact solutions for quantum Zakharov equations. The main idea of this method is to take full advantage of the generalized Riccati equation which has more new solutions. It seems that the Exp-function method is more effective and simple than other methods, and a lot of solutions can be obtained in the same time. In addition, this method is also computerizable, which allows us to perform complicated and tedious algebraic calculation on a computer.

In view of the Exp-function method, we give a very simple and straightforward method for nonlinear evolution equations arising in mathematical physics. We would like to make some important remarks on the method:

1) The method leads to both generalized solitary solutions and periodic solutions.

2) The expression of the Exp-function is more general than the sinh-function and the tanh-function, so we can find more general solutions with the Exp-function method.

3) The solution procedure, using Maple or Mathematica, is of utter simplicity.

4) The Exp-function method can be employed in both the straightforward way and the sub-equation way. But we suggest that it is better to use this method directly, not only for its convenience, but also because it is sometimes possible to lose some information and solutions if one applies it in the sub-equation way.

Acknowledgements

The authors would like to express their great thankfulness to Prof. S. A. El-Wakil for his encouragement and supervision and to the referees for their useful comments and discussion.

[1] S. A. El-Wakil, E. M. Abulwafa, A. Elhanbaly, and M. A Abdou, Chaos, Solitons and Fractals **33**, 1512 (2007).
[2] S. A. El-Wakil, M. A. Abdou, and A. Elhanbaly, Phys. Lett. A **353**, 40 (2006).
[3] M. A. Abdou and A. Elhanbaly, Commun. Nonlinear Sci. Numerical Simulation **12**, 1229 (2007).
[4] S. A. El-Wakil and M. A. Abdou, Chaos, Solitons and Fractals **36**, 343 (2008).
[5] S. A. El-Wakil and M. A. Abdou, Chaos, Solitons and Fractals **31**, 840 (2007).
[6] S. A. El-Wakil and M. A. Abdou, Nonlinear Analysis **68**, 235 (2008).
[7] M. A. Abdou, Chaos, Solitons and Fractals **31**, 95 (2007).
[8] M. A. Abdou, J. Nonlinear Dynamics **52**, 277 (2007).
[9] J.-H. He, Int. J. Modern Phys. B **20**, 1141 (2006).
[10] J. H. He, Non-Perturbative Method for Strongly Nonlinear Problems, dissertation.de – Verlag im Internet, Berlin 2006.
[11] M. A. Abdou and A. A. Soliman, Physica D **211**, 1 (2005).
[12] M. A. Abdou, Phys. Lett. A **366**, 61 (2007).
[13] M. A. Abdou, J. Comput. Appl. Math. **214**, 202 (2008).
[14] M. A. Abdou, Int. J. Nonlinear Sci. **5**, 1 (2008).
[15] E. M. Abulwafa, M. A. Abdou, and A. A. Mahmoud, Chaos, Solitons and Fractals **29**, 313 (2006).
[16] M. Abdou and A. Elhanbaly, Phys. Scr. **73**, 338 (2006).
[17] A. Elhanbaly and M. A. Abdou, Appl. Math. Comput. **182**, 301 (2006).
[18] S. A. El-Wakil, M. A. Abdou, and A. Elhanbaly, Appl. Math. Comput. **1827**, 313 (2006).
[19] S. A. El-Wakil and M. A. Abdou, Chaos, Solitons and Fractals **33**, 513 (2007).
[20] M. A. Abdou and A. Elhanbaly, Phys. Scr. **73**, 338 (2006).
[21] M. L. Wang, Y. M. Wang, and J. L. Zhang, Chin. Phys. **12**, 1341 (2003).
[22] J.-H. He and M. A. Abdou, Chaos, Solitons and Fractals **34**, 1421 (2007).
[23] S. A. El-Wakil, M. Madkour, and M. A. Abdou, Phys. Lett. A **369**, 62 (2007).
[24] S. Zhang, Phys. Lett. A **372**, 1873 (2008).
[25] S. A. El-Wakil, M. A. Abdou, and A. Hendi, Phys. Lett. A **372**, 830 (2008).
[26] M. A. Abdou, Nonlinear Dynamics **52**, 1 (2008).
[27] S. A. El-Wakil and M. A. Abdou, Phys. Lett. A **358**, 275 (2006).
[28] M. A. Abdou and S. Zhang, Commun. Nonlinear Sci. Numerical Simulation **14**, 2 (2009).
[29] S. A. El-Wakil and M. A. Abdou, Z. Naturforsch. **63a**, 385 (2008).
[30] M. A. Abdou, Nonlinear Dynamics **52**, 95 (2008).
[31] S. A. El-Wakil and M. A. Abdou, Nonlinear Dynamics **51**, 585 (2008).
[32] S. A. El-Wakil and M. A. Abdou, Chaos, Solitons and Fractals **33**, 513 (2007).
[33] S. A. El-Wakil, M. Madkour, and M. A. Abdou, Phys. Lett. A **353**, 40 (2006).
[34] V. E. Zakharov, Zh. Eksp. Theor. Fiz. **62**, 1745 (1972).
[35] V. E. Zakharov, Sov. Phys. JETP **35**, 908 (1972).